I part

1. A real current generator, with the current $I_{g1} = 20\ \text{mA}$ of the internal resistance $R_1 = 1\ \text{M}\Omega$, as well as the second real current generator, with the current I_{g2} of the internal resistance $0 < R_2 < R_1$ make together a circuit, as it is shown in Fig. 1. Calculate the maximal possible power absorbed by the second current generator (I_{g2}, R_2) if I_{g2} can be varied in a wide range.

Solution:
a) $P_{2max} = 5\ \text{W}$
b) $P_{2max} = 10\ \text{W}$
c) $P_{2max} = 50\ \text{W}$
d) $P_{2max} = 100\ \text{W}$
e) none of the above

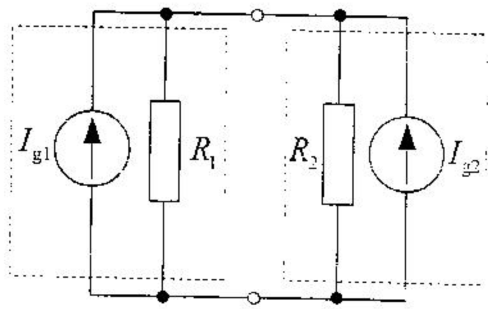


Figure 1.

I part

2. Consider an infinite resistive network, shown in Fig. 2. Calculate the equivalent resistance between points A and B if $R_1 = 1\Omega$ and $R_2 = 4\Omega$.

Solution:
a) $R_{AB} = 1\ \Omega$
b) $R_{AB} = 2\ \Omega$
c) $R_{AB} = 3\ \Omega$
d) $R_{AB} = \sqrt{6}\ \Omega$
e) none of the above

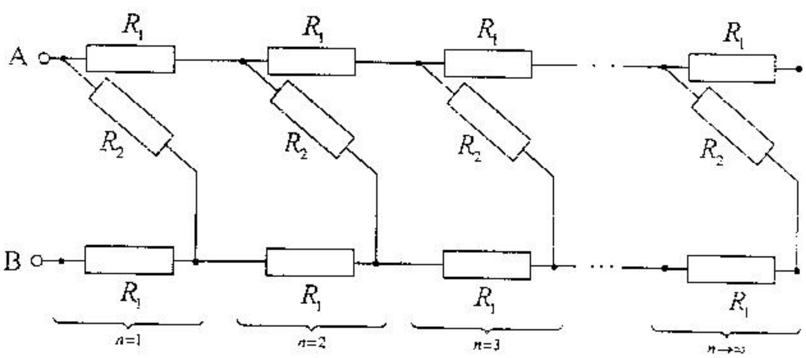


Figure 2.

II part

3. In the DC circuit, shown in Fig. 3, the given parameters are: $E_1 = 15$ V, $R_1 = 4$ kΩ, $R_2 = 4$ kΩ, $R_3 = 1$ kΩ and $R_4 + R_5 = 3$ kΩ. In the case when the switch Π is open the voltmeter with the internal resistance $R_V = 10$ kΩ shows the voltage $U_{21} = 15$ V. Calculate the current I of the resistor R_3 in the case when the switch Π is closed, if the voltmeter measures $U'_{21} = 20$ V?

Solution: **a)** $I = 2.75$ A
b) $I = 3.75$ A
c) $I = -3.75$ A
d) $I = 4$ A
e) none of the above

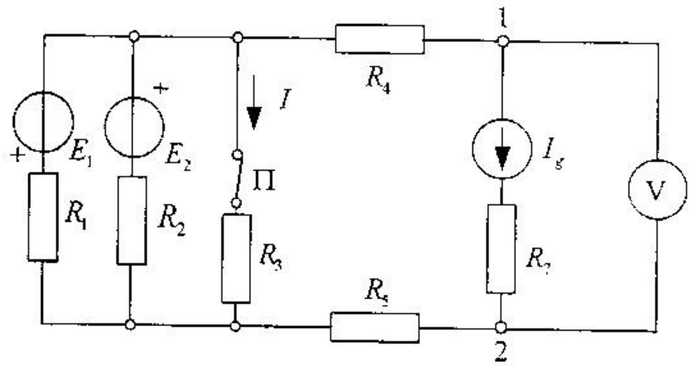


Figure 3.

II part

4. In the DC circuit shown in Fig. 4, the given parameters are $E_3 = 10$ V, $R_1 = R_5 = 200$ Ω, $R_2 = 100$ Ω, $R_3 = 300$ Ω, $R_4 = 400$ Ω and $E_1 = E_2$. Calculate the power of the ideal voltage generator with the electromotive force E_3.

Solution: **a)** $P_{E3} = 200$ mW
b) $P_{E3} = 300$ mW
c) $P_{E3} = 400$ mW
d) $P_{E3} = 500$ mW
e) none of the above

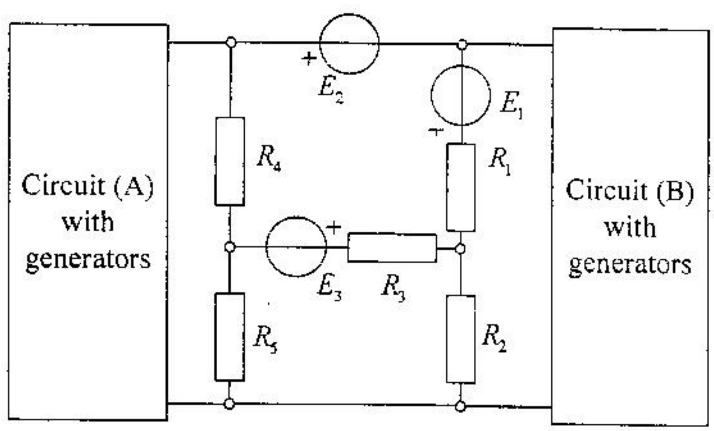


Figure 4.

III part

5. In the DC circuit (Fig. 5), when the switch Π is open, the energy stored in the capacitor C_1 is $W_{C1} = 36\ \mu J$ and the energy stored in the capacitor C_2 is $W_{C2} = 0.1\ \mu J$. What is the energy W'_{C2} stored in the capacitor C_2 in the case when the switch Π is closed? The known parameters of the circuit are: $E_1 = 12\ V$, $R_1 = 2\ k\Omega$, $R_4 = 200\ \Omega$, $C_1 = 2\ \mu F$ and $C_2 = 5\ \mu F$.

Solution:

a) $W'_{C2} = 0.1\ \mu J$
b) $W'_{C2} = 0.2\ \mu J$
c) $W'_{C2} = 0.4\ \mu J$
d) $W'_{C2} = 0.8\ \mu J$
e) none of the above

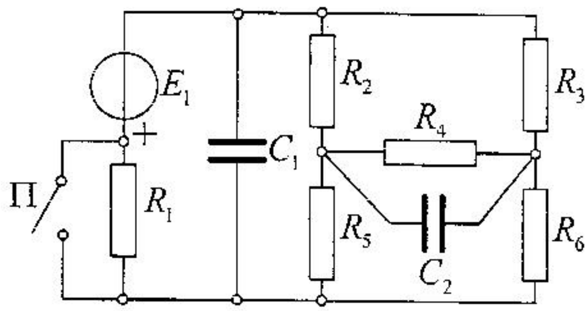


Figure 5.

III part

6. A generator in the shape of a thin disk is shown in Fig. 6. The basis of the disk is a circle of the radius $r = 5$ mm, and the height of the disk is $d = 0{,}5$ mm ($d << r$). There is an impressed homogeneous field of the intensity $E_i = 3$ kV/m in the generator. The vector $\vec{E}_i$ is perpendicular to the bases of the disk, as it is shown in Fig. 6. The specific conductance of the material in generator is $\sigma = 10$ S/m, while the losses in the connecting conductors are negligible. The generator is connected in a circuit so that there is a DC current $I = 100$ mA in the connecting conductors. Calculate the intensity of the electric field in the generator.

Solution:

a) $E = 0$
b) $E = 2.873$ kV/m
c) $E = 3$ kV/m
d) $E = 3.127$ kV/m
e) none of the above

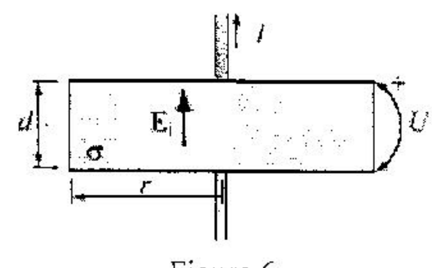


Figure 6.

IV part

7. A spherical capacitor has two dielectrics, as it is shown in Fig. 7. The dimensions of the capacitor are $0 < a < b$, $b = 7$ mm, $c = 10$ mm, the relative permittivity of the first dielectric is $\varepsilon_{r1} = 8$ and the relative permittivity of the second dielectric is $\varepsilon_{r2} = 4$. The first dielectric is in a solid state, while the second is in a fluid state. The capacitor is connected to the voltage source, and after that the switch Π is opened. While the switch is open, the second dielectric pours out of the capacitor through a small hole at the outer electrode. The volume previously filled with the second dielectric is substituted with the air, and the breakdown occurs in the air. Calculate the change in the capacitor voltage from the time of opening of the switch Π until the moment of the breakdown. The intensity of the breakdown electric field in the air is $E_{kr} = 30$ kV/cm.

Solution:
- **a)** $\Delta U = 4725$ V
- **b)** $\Delta U = 3725$ V
- **c)** $\Delta U = 2725$ V
- **d)** $\Delta U = 1725$ V
- **e)** none of the above

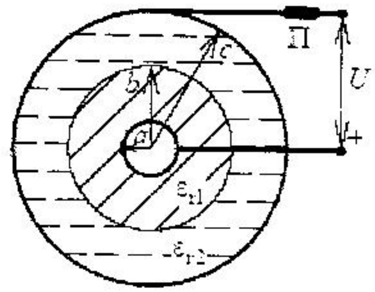


Figure 7.

IV part

8. A plate capacitor, filled with air, is shown in Fig. 8. The distance between electrodes is d. The capacitor is connected to the voltage source, and then the source is removed. What is the ratio $k = F(d)/F(d/2)$ of the intensity of total forces on one plate (electrode) of the capacitor, in the case when the distance between plates is d and when the distance is $d/2$? The end effects can be neglected.

Solution:
- **a)** $k = 1$
- **b)** $k = 0.25$
- **c)** $k = 2$
- **d)** $k = 4$
- **e)** none of the above

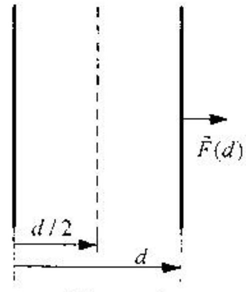


Figure 8.

V part

9. In a very long hollow conductor with the square cross-section of the side a and thickness $\delta << a$ (Fig. 9) there is total DC current I. The permeability is μ_0 everywhere. Find the intensity of the magnetic induction $\vec{B}$ at the point M which is in the middle of the conductor cross-section.

Solution:
a) $B = 0$
b) $B = \dfrac{\mu_0 I}{a}$
c) $B = \dfrac{\mu_0 I}{2a}$
d) $B = \dfrac{\mu_0 I}{4a}$
e) none of the above

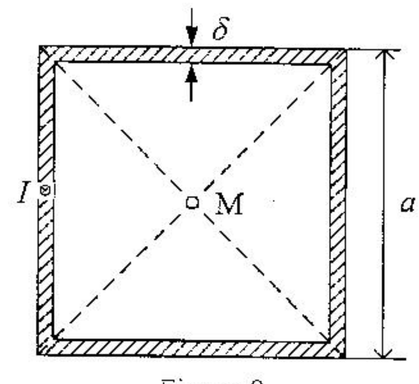


Figure 9.

V part

10. On the whole surface of a styrofoam sphere of the radius a, uniformly and densely one next to the other are stacked N turns of thin wire with current $i(t) = I_m \sin \omega t$, as it is shown in Fig. 10. The sphere is placed in time-varying magnetic field of the magnetic induction $\vec{B}(t) = \dfrac{B_m}{2}(\vec{i}_x + \vec{i}_z\sqrt{3})\cos \omega t$. What is the average magnetic torque within one period of the specified current?

Solution:
a) $\vec{M} = \dfrac{\pi^2 a^2 N I_m B_m}{8}\vec{i}_y$
b) $\vec{M} = \dfrac{\pi a^2 N I_m B_m}{8}\vec{i}_y$
c) $\vec{M} = -\dfrac{\pi a^2 N I_m B_m}{8}\vec{i}_y$
d) $\vec{M} = 0$
e) none of the above

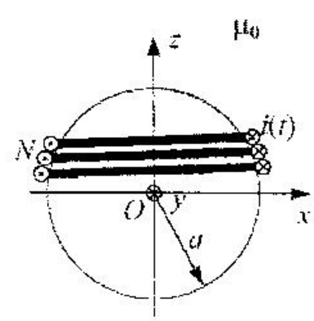


Figure 10.

VI part

11. In the plane of a very long thin-wire conductor with DC current $I = 100\,\text{A}$, at the distance $b = 33\,\text{cm}$, is placed an axis along which rotates a metallic rod, of length $a = 16.5\,\text{cm}$, with constant angular velocity $\omega = 500\pi$ rad/s , as it is shown in Fig. 11. Calculate the maximal induced electromotive force e_{max} in the metallic rod. The surrounding medium is a vacuum.

Solution:

a) $e_{max} = 4$ mV
b) $e_{max} = 3$ mV
c) $e_{max} = 2$ mV
d) $e_{max} = 0.95$ mV
e) none of the above

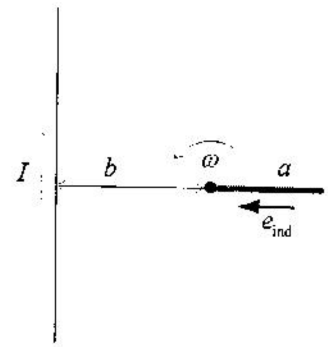


Figure 11.

VI part

12. Within thin torus made of ferromagnetic material, of mid-line length $l = 100\,\text{mm}$ and circular cross-section area $S = 1\,\text{cm}^2$, there is remnant magnetization. The vector of magnetization is tangential on the mid-line of the torus, and the magnetization intensity is $M = 1.5/\pi 10^5$ A/m at all points. There are $N = 300$ turns of wire uniformly and densely, one next to the other, stacked at the surface of the torus. The total resistance of the wire is $R = 6\,\Omega$. The ideal contact K rotates with the constant angular velocity ω as it is shown in Fig. 12. During the time for which the contact travels the angle $\varphi = 4\pi/3$, what is the total charge q indicated by the ballistic galvanometer (B.G.) with respect to the given reference direction shown on Fig. 12. The internal resistance of the ballistic galvanometer is $R_g = 1\,\Omega$. The surrounding medium is a vacuum.

Solution:

a) $q = -240$ μC
b) $q = 240$ μC
c) $q = 150$ μC
d) $q = 0$
e) none of the above

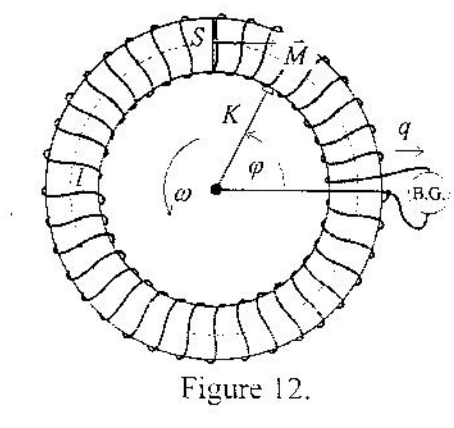


Figure 12.

VII part

13. In the AC circuit shown in Fig. 13 given are $e(t) = 2\cos\omega t$ V, $i_g(t) = I_m \sin\omega t$ A and $1/\omega C = 12.5$ kΩ. The voltage $u(t)$ has phase delay of $\pi/2$ with respect to the electromotive force. Calculate the average power P of the ideal generator of the electromotive force $e(t)$.

Solution:
- a) $P = 1.6$ W
- b) $P = 16$ mW
- c) $P = 160$ μW
- d) $P = 16$ W
- e) none of the above

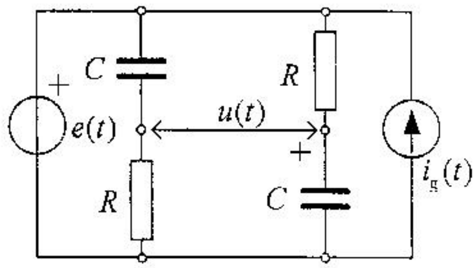


Figure 13.

VII part

14. In the part of AC circuit shown in Fig. 14 given are rms voltages $U_{13} = U_{32} = 2$ V, electromotive force $E_1 = 2\sqrt{3}$ V and (the modulus of) impedance $Z_1 = 2\sqrt{3}$ Ω. The voltage $u_{32}(t)$ is phase-shifted with respect to electromotive force $e_1(t)$ for $\pi/6$, while electromotive force $e_1(t)$ is phase-shifted with respect to $u_{13}(t)$ for $-\pi/2$. Calculate the current of the ideal ammeter I_{12}.

Solution:
- a) $I_{12} = 1$ A
- b) $I_{12} = \dfrac{\sqrt{3}}{3}$ A
- c) $I_{12} = 2\sqrt{3}(4-2\sqrt{3})$ A
- d) $I_{12} = \sqrt{3}$ A
- e) none of the above

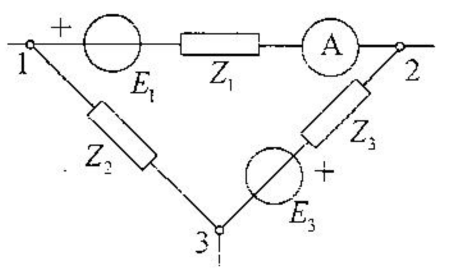


Figure 14.

VIII part

15. In the AC circuit shown in Fig. 15, when the switch Π is closed, the complex current through the switch is $\underline{I} = (3 - \mathrm{j})$ A. Calculate the complex voltage $\underline{U}_{AB}$, when the switch Π is open, if the complex power of the second current generator is $\underline{S}_{g2} = (10 + \mathrm{j}6)$ VA. Given are: $\underline{Z} = (2 - \mathrm{j}2)\ \Omega$, $\underline{Z}_1 = (1 + \mathrm{j}2)\ \Omega$, $\underline{Z}_2 = \underline{Z}_4 = (2 + \mathrm{j}4)\ \Omega$, $\underline{Z}_3 = (2 - \mathrm{j}4)\ \Omega$, $\underline{I}_{g1} = (0.5 + \mathrm{j}1.5)$ A and $\underline{E} = (7 + \mathrm{j}3.5)$ V

Solution:
a) $\underline{U}_{AB} = -(8.5 + \mathrm{j}4.5)$ V
b) $\underline{U}_{AB} = -(16.5 + \mathrm{j}0.5)$ V
c) $\underline{U}_{AB} = -(4.5 - \mathrm{j}3.5)$ V
d) $\underline{U}_{AB} = -(9.5 + \mathrm{j}6.5)$ V
e) none of the above

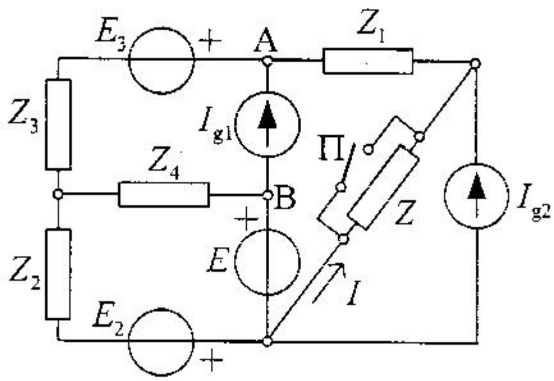


Figure 15.

VIII part

16. Apparent powers of the loads shown in Fig. 16 are $S_1 = 1$ VA и $S_2 = 7$ VA. The voltage U_1 is phase-shifted with respect to current I for $\pi/4$, while the voltage U_2 is phase-shifted with respect to the current I for $-\pi/4$. Calculate the power factor $\cos\varphi$ these two loads connected in series.

Solution:
a) $\cos\varphi = 0{,}5$
b) $\cos\varphi = 0.6$
c) $\cos\varphi = \sqrt{2}/2$
d) $\cos\varphi = 0.8$
e) none of the above

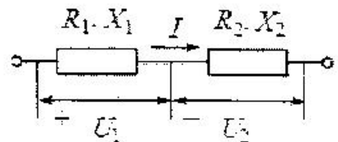


Figure 16.

IX part

17. In the AC circuit shown in Fig. 17 know are the parameters *R*, *L* и *C*. The rms current of the current generator is constant, while the frequency (*f*) can be changed. Find the frequency *f* at which the rms current I_L is maximal.

Solution: **a)** $f = \dfrac{1}{2\pi\sqrt{LC}}$

b) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{1}{2R^2C^2}}$

c) $f = \dfrac{1}{2\pi RC}$

d) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{1}{R^2C^2}}$

e) none of the above

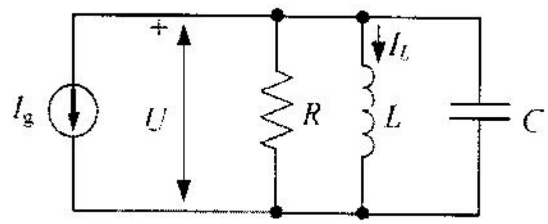


Figure 17.

IX part

18. The circuit shown in Fig. 18 is powered **by** s simple oscillating voltage generator of the complex electromotive force $\underline{E}$. Calculate the frequency ω at which the voltage $\underline{U}$ is in-phase with the electromotive force $\underline{E}$. Given are: $L = 0.1$ mH, $C = 10$ nF and $k = 1$.

Solution: **a)** $\omega = 10^6\ \mathrm{s}^{-1}$

b) $\omega = \dfrac{1}{2}\cdot 10^6\ \mathrm{s}^{-1}$

c) $\omega = \sqrt{2}\cdot 10^6\ \mathrm{s}^{-1}$

d) $\omega = \dfrac{\sqrt{2}}{2}\cdot 10^6\ \mathrm{s}^{-1}$

e) none of the above

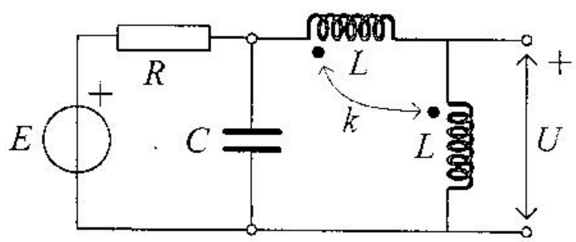
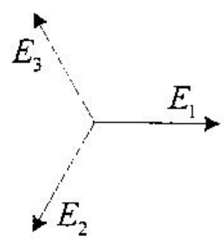


Figure 18.

X part

19. Generators of simple oscillating electromotive forces E_1, E_2 and E_3 make direct symmetrical three-phase system. Asymmetric three-phase load is connected to this three-phase generator as it is shown in Fig. 19. Given are: resistance of the resistor $R = 100\ \Omega$, impedance of inductor $Z_L = 200\ \Omega$ and impedance of capacitor $Z_C = 100\ \Omega$. The switch Π is open and the circuit is in the AC regime. Then, the switch is closed. In the new AC regime of the circuit, change in of the complex voltage $\underline{U}_{AB}$ with respect to the previous regime is $\Delta \underline{U}_{AB} = \mathrm{j}600$ V? Calculate the complex power of the three-phase load when the switch Π is open.

Solution:
a) $\underline{S} = 1200(1-\mathrm{j})$ VA
b) $\underline{S} = 1200(1+\mathrm{j})$ VA
c) $\underline{S} = 600(1+\mathrm{j}\sqrt{3})$ VA
d) $\underline{S} = 600(1-\mathrm{j}\sqrt{3})$ VA
e) none of the above



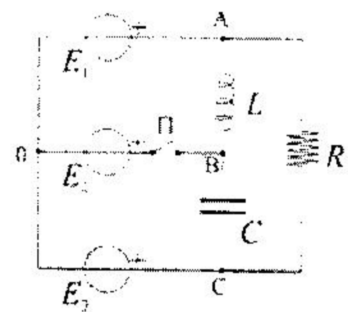


Figure 19.

X part

20. Symmetrical three-phase load, shown in Fig. 20, is connected to a three-phase network of symmetrical phase voltages, of the angular frequency $\omega = 100\ \pi$. Given are: $R = \omega L = 5\omega L_1 = 100\ \Omega$. Calculate the minimal capacitance C of capacitors, so that the power-factor at the feeding line is maximal.

Solution:
a) $C = \dfrac{600}{\pi}$ μF
b) $C = \dfrac{200}{\pi}$ μF
c) $C = \dfrac{100}{\pi}$ μF
d) $C = \dfrac{20}{\pi}$ μF
e) none of the above

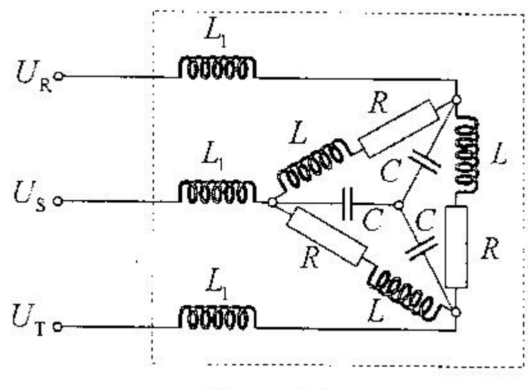


Figure 20.